MONTE CARLO STUDY OF THE
METAMAGNET ISING MODEL IN THREE
DIMENSIONS IN A RANDOM FIELD

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The metamagnet is a kind of anisotropic antiferromagnetic materials (compound). This materials, such as FeCl$_2$, FeBr$_2$, Ni(NO$_3$)$_2$2H$_2$O, and DAG (Dysprosium Aluminium Garnet), own a structure of alternate layers with ferromagnetic couplings between the spins in each layer but antiferromagnetic couplings between adjacent layers. In reason of that structure it showed a behaviour no-orthodox, some times it behaves as a ferromagnetic, other times as an antiferromagnetic. To field null or to sufficiently low field, the Ising metamagnetic behaves as an Ising antiferromagnetic with a phase transition of second order. However, for fields sufficiently big, the phase transition changes abruptly from second to first order, this changes occur at the tricritical temperature[1-3].
Metamagnet ideal can be considered as a set of spins with uniaxial anisotropy ferromagnetic interactions within each \( (J_1 > 0) \) and antiferromagnetic interaction \( (J_2 < 0) \). The hamiltonian model for spin-1/2 is given by:

\[
\mathcal{H} = - \sum_{\langle i,j \rangle} J_1 \sigma_i \sigma_j - \sum_{\langle i,j \rangle} J_2 \sigma_i \sigma_k - \sum_{i=1}^{N} (h - h_i) \sigma_i \tag{1}
\]

where the first is executed on all pairs of spin nearest-neighbors on same plane and second sum run over all pairs of spin nearest-neighbor in parallel planes, \( h \) is the strength of the external uniform magnetic field and \( h_i \) is the random magnetic field which obeys the bimodal distribution given by:

\[
P(h_i) = \frac{1}{2} [\delta(h_i - h_r) + \delta(h_i + h_r)] \tag{2}
\]

where \( h_r \) is the strength of the random field.
To study this system we employed Monte Carlo simulation technique [2] by using the algorithm of Glauber in a cubic lattice of linear size $L$ with values ranging from 16 to 42 and with periodic boundary conditions. To reach the equilibrium state we take, for guarantee, at least $2 \times 10^4$ Monte Carlo steps (MCs) for all the lattice sites we studied and more $3 \times 10^4$ MCS to estimate the average values of the quantities of interest. In our work we consider one MCs equivalent $L^3$ trials for change the state of a spin of the lattice.
We calculated the sublattice magnetization per spin belong to the different planes by using

\[
m_A = \left[ \frac{2}{N} \left\langle \sum_{i \in A} \sigma_i \right\rangle \right], \tag{3}
\]

\[
m_B = \left[ \frac{2}{N} \left\langle \sum_{i \in B} \sigma_i \right\rangle \right], \tag{4}
\]

The transition lines of the phase diagram were obtained from the staggered magnetization, \( m_s \) and magnetization \( m \). Thus we calculate \( m_s = m_A - m_B \) and \( m = m_A + m_B \) that are our parameters of order for antiferromagnetic and ferromagnetic phase, respectively. In the above equations \([\cdots]\) denotes the average over the disorder and \( \langle \cdots \rangle \) denotes the thermal average.
Results and Discussion

The Figure 1 we observed the magnetization vs. temperature behavior for different external field, in the Figure 2 the magnetization vs field for different temperature presenting transitions of first and second order. The Figure 3 illustrate the complete phase diagram, for two selected values of the random field $h_r$, showing the continuous and discontinuous transition lines. The tricritical point, which is indicated by an open square, joins these two lines. The points of the phase diagram are obtained from the knowledge of the point of maximum of the curve for the susceptibility. In this work the location of the tricritical point was achieved through the disappearance of hysteresis [4-6]. For a fixed value of temperature, we drew the magnetization curves for increasing and decreasing values of the magnetic field (Fig. 4 and 5).
Figure 1: Curves of magnetization $m$ and staggered magnetization $m_s$ for different values of external field.

Figure 2: Curves of magnetization $m$ versus external field for different values of temperature.
Results and Discussion

Figure 3: Phase diagram of metamagnet in a cubic lattice in the plane $t - h$ for $h_r = 0$ and $h_r = 1$. 

- Discontinuous transition
- Continuous transition
- Ponto Tricritico
Results and Discussion

Figure 4: Hysteresis curve for $T < T_{\text{tricritical}}$

Figure 5: Hysteresis curve for $T > T_{\text{tricritical}}$
Results and Discussion

Figure 6: Curves of magnetization $m$ vs. external field for different values of random field $h_r$. 

$T=1$

Magnetization

$h = 0$
$h = 2$
$h = 4$

$h_r$

$0,0 0,5 1,0 1,5 2,0 2,5 3,0 3,5 4,0 4,5 5,0$

$h$

$0,0 0,2 0,4 0,6 0,8 1,0$
In summary, the present Monte Carlo simulations for a metamegnet Ising model in a random and uniform field show that the phase diagram in the plane uniform field $h$ versus temperature present continuous and first-order transition lines separated by tricritical points.


